Problem Set Solutions (Based on Provided Document)

Problem 14.1

(a) How many watts of power are contained in the light from a 1000 lumen video projector?

Solution: The document states that an ideal source of white light over the visible range would produce about 200 lumens (lm) per watt (W) of power[cite: 32]. However, typical incandescent sources are much lower (10-20 lm/W [cite: 33]), and efficient lamps like sodium vapor approach 200 lm/W[cite: 35]. Video projectors use various lamp technologies, often less efficient than the ideal 200 lm/W due to filtering for color, etc. Assuming a relatively efficient projector might operate at, say, half the ideal efficiency for white light as a rough estimate:

$$\text{Efficiency} \approx 100\,\text{lm/W}$$

Then, the power P for a $L = 1000$ lm projector is:

$$P = \frac{L}{\text{Efficiency}} = \frac{1000\,\text{lm}}{100\,\text{lm/W}} = 10\,\text{W}$$

This is an estimate; the actual power depends heavily on the specific projector technology and efficiency. Using the ideal efficiency yields $P = 1000/200 = 5$ W[cite: 32]. A typical 75W bulb produces 1200 lm, giving 16 lm/W[cite: 33]; using this efficiency gives $P = 1000/16 \approx 62.5$ W. The value varies greatly. Let's use the ideal source value as a lower bound.

Answer: Using the ideal efficiency value provided in the text[cite: 32], the power is $1000\,\text{lm}/200\,\text{lm/W} = 5\,\text{W}$. Using efficiencies of real sources suggests it could be significantly higher (e.g., 10-60 W)[cite: 33, 34, 35].

(b) What spatial resolution is needed for the printing of a page in a book to match the eye's limit?

Solution: The document states that the eye can resolve a spatial frequency of 60 cycles per degree[cite: 81]. To convert this to a resolution like dots per inch (dpi), we need a viewing distance. Assume a standard reading distance, $d = 30$ cm (approx 1 foot).

One degree at distance d subtends a length $s = d\tan(1^\circ)$.

$$s = (30\,\text{cm})\tan(1^\circ) \approx (30\,\text{cm}) \times (0.01745) \approx 0.524\,\text{cm}$$

The eye resolves 60 cycles in this angle/length. One cycle corresponds to a pair of lines (e.g., black and white), so it contains 2 "dots" or features. The minimum resolvable feature size Δx corresponds to half a cycle length at the limit.

$$\text{Length per cycle} = \frac{s}{60} = \frac{0.524\,\text{cm}}{60} \approx 0.00873\,\text{cm}$$

$$\text{Minimum feature size } \Delta x = \frac{1}{2} \times (\text{Length per cycle}) \approx 0.00436\,\text{cm}$$

Resolution in dots per cm (dpcm) is $1/\Delta x$.

$$\text{Resolution (dpcm)} = \frac{1}{0.00436\,\text{cm}} \approx 229\,\text{dpcm}$$

Converting to dots per inch (dpi), knowing 1 inch = 2.54 cm:

$$\text{Resolution (dpi)} = 229\,\text{dpcm} \times 2.54\,\frac{\text{cm}}{\text{inch}} \approx 582\,\text{dpi}$$

Answer: A spatial resolution of approximately 600 dpi is needed to match the eye's limit at a typical reading distance.

Problem 14.2

(a) What is the peak wavelength for black-body radiation from a person? From the cosmic background radiation at 2.74 K?

Solution: The peak wavelength λ_{peak} of black-body radiation is given by Wien's Displacement Law:

$$\lambda_{\text{peak}} = \frac{b}{T}$$

where $b \approx 2.898 \times 10^{-3}\,\mathrm{m} \cdot \mathrm{K}$ is Wien's displacement constant, and T is the absolute temperature in Kelvin. While the document provides Planck's Law (Eq. 13.5)[cite: 7], it doesn't explicitly state Wien's Law, which is derived from it.

Person: Assume a normal body temperature $T_{\text{person}} = 37°\mathrm{C} = (37 + 273.15)\,\mathrm{K} \approx 310\,\mathrm{K}$.

$$\lambda_{\text{peak, person}} = \frac{2.898 \times 10^{-3}\,\mathrm{m} \cdot \mathrm{K}}{310\,\mathrm{K}} \approx 9.35 \times 10^{-6}\,\mathrm{m} = 9.35\,\mu\mathrm{m}$$

This falls in the infrared region.

Cosmic Background Radiation (CMB): The temperature is given as $T_{\text{CMB}} = 2.74\,\mathrm{K}$[cite: 324].

$$\lambda_{\text{peak, CMB}} = \frac{2.898 \times 10^{-3}\,\mathrm{m} \cdot \mathrm{K}}{2.74\,\mathrm{K}} \approx 1.058 \times 10^{-3}\,\mathrm{m} = 1.058\,\mathrm{mm}$$

This falls in the microwave region.

Answer: Peak wavelength for a person is $\approx 9.35\,\mu\mathrm{m}$ (infrared). Peak wavelength for CMB at 2.74 K is $\approx 1.06\,\mathrm{mm}$ (microwave).

(b) Approximately how hot is a material if it is "red-hot"?

Solution: "Red-hot" implies the material is emitting enough thermal radiation in the red part of the visible spectrum ($\lambda \approx 650 - 750$ nm) to be perceived as red. Using Wien's Law ($\lambda_{\text{peak}} = b/T$) again, we can estimate the temperature where the peak emission is red.

$$T = \frac{b}{\lambda_{\text{peak}}} = \frac{2.898 \times 10^{-3}\,\mathrm{m} \cdot \mathrm{K}}{700 \times 10^{-9}\,\mathrm{m}} \approx 4140\,\mathrm{K}$$

However, an object appears red even when the peak is in the infrared, as long as the tail of the Planck distribution (Eq. 13.5[cite: 7], Figure 13.1 [cite: 8]) extends significantly into the visible red. Objects typically start glowing visibly red around 500-600 °C (approx 800-900 K). At these temperatures, the peak is still deep in the infrared ($\sim 3 - 4\,\mu\mathrm{m}$), but there's enough emission at the red end of the visible spectrum.

Answer: A material typically starts glowing visibly red around 800-900 K (approx 500-600 °C). The peak emission at this point is still in the infrared.

(c) Estimate the total power thermally radiated by a person.

Solution: The total power per area R radiated by a black body is given by the Stefan-Boltzmann Law (Eq. 13.7)[cite: 13]:

$$R = \sigma T^4$$

where $\sigma = 5.67 \times 10^{-8}\,\mathrm{W/(m^2 \cdot K^4)}$ is the Stefan-Boltzmann constant and T is the absolute temperature[cite: 13]. For real surfaces, this is corrected by emissivity ϵ ($R = \epsilon\sigma T^4$), where $\epsilon = 1$ for an ideal black body[cite: 14]. Human skin has high emissivity in the infrared, $\epsilon \approx 0.95 - 0.98$.

Assume body temperature $T = 310\,\mathrm{K}$ (from part a), surface area $A \approx 1.7\,\mathrm{m}^2$, and emissivity $\epsilon \approx 0.95$. The total power radiated P_{rad} is:

$$P_{\text{rad}} = \epsilon\sigma A T^4$$

$$P_{\text{rad}} \approx (0.95)(5.67 \times 10^{-8}\,\mathrm{W/(m^2 \cdot K^4)})(1.7\,\mathrm{m}^2)(310\,\mathrm{K})^4$$

$$P_{\text{rad}} \approx (0.95)(5.67 \times 10^{-8})(1.7)(9.235 \times 10^9)\,\mathrm{W} \approx 846\,\mathrm{W}$$

This is the power radiated. The net power loss also depends on the power absorbed from the surroundings. If the surrounding temperature is T_{surr}, the power absorbed is $P_{\text{abs}} = \epsilon\sigma A T_{\text{surr}}^4$. Assuming room temperature $T_{\text{surr}} = 20°\mathrm{C} = 293\,\mathrm{K}$:

$$P_{\text{abs}} \approx (0.95)(5.67 \times 10^{-8})(1.7)(293\,\mathrm{K})^4 \approx 675\,\mathrm{W}$$

The net radiative power loss is $P_{\text{net}} = P_{\text{rad}} - P_{\text{abs}} \approx 846 - 675 = 171\,\mathrm{W}$.

Answer: The total power thermally radiated by a person at 310 K is approximately 850 W. The net radiative power loss in a 20 °C environment is roughly 170 W.

Problem 14.3

We use the Jones calculus formalism described on pages 13-14[cite: 169]. Let the input light be linearly polarized along the x-axis: $\vec{E}_{\text{in}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. The birefringent material has its optical axes rotated by an angle θ relative to the lab (x,y) axes. The transformation is $\vec{E}_{\text{out}} = R(-\theta)B(d)R(\theta)\vec{E}_{\text{in}}$[cite: 178], where $R(\theta)$ is the rotation matrix [cite: 174] and $B(d)$ is the birefringence matrix[cite: 177], ignoring the overall phase $e^{-i\sigma}$:

$$R(\theta) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \quad B(d) = \begin{pmatrix} e^{-i\delta} & 0 \\ 0 & e^{i\delta} \end{pmatrix}$$

The phase difference term is $\delta = (n_{\text{slow}} - n_{\text{fast}})\frac{\omega d}{2c} = (n_s - n_f)\frac{\pi d}{\lambda}$[cite: 176, 177]. For calcite, $n_s = 1.658$ and $n_f = 1.486$[cite: 167], so $\Delta n = n_s - n_f = 0.172$. The wavelength is $\lambda \approx 600\,\text{nm} = 600 \times 10^{-9}\,\text{m}$[cite: 328].

(a) Find a thickness and an orientation for a birefringent material that rotates a linearly polarized wave by 90°. What is that thickness for calcite with visible light ($\lambda \sim 600$ nm)?

Solution: To rotate linear polarization by 90° (e.g., from x-polarized to y-polarized, $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$), we need a half-wave plate ($\delta = \pi/2$, corresponding to a phase difference of $2\delta = \pi$). The plate must be oriented at $\theta = 45°$ relative to the input polarization. Let's verify: $\theta = 45°$, $\cos\theta = \sin\theta = 1/\sqrt{2}$. $R(45°) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$. $R(-45°) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$. For $\delta = \pi/2$, $B(d) = \begin{pmatrix} e^{-i\pi/2} & 0 \\ 0 & e^{i\pi/2} \end{pmatrix} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}$. The transformation matrix $M = R(-45°)B(d)R(45°)$:

$$M = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} -i & -i \\ -i & i \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & -2i \\ -2i & 0 \end{pmatrix} = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}$$

Applying to $\vec{E}_{\text{in}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$:

$$\vec{E}_{\text{out}} = M\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -i \end{pmatrix}$$

This is y-polarized light (with a phase shift). It has been rotated by 90°. The thickness d required for $\delta = \pi/2$ is found from $\delta = (n_s - n_f)\frac{\pi d}{\lambda}$:

$$\frac{\pi}{2} = (n_s - n_f)\frac{\pi d}{\lambda} \implies d = \frac{\lambda}{2(n_s - n_f)}$$

For calcite and $\lambda = 600$ nm:

$$d = \frac{600 \times 10^{-9}\,\text{m}}{2 \times (0.172)} \approx 1.744 \times 10^{-6}\,\text{m} = 1.74\,\mu\text{m}$$

Answer: A half-wave plate ($\delta = \pi/2$) oriented at $\theta = 45°$ to the input polarization rotates it by 90°. For calcite at $\lambda = 600$ nm, the required thickness is $d = \lambda/(2\Delta n) \approx 1.74\,\mu$m.

(b) Find a thickness and an orientation that converts linearly polarized light to circularly polarized light, and evaluate the thickness for calcite.

Solution: To convert linear polarization to circular polarization, we need a quarter-wave plate ($\delta = \pi/4$, phase difference $2\delta = \pi/2$). The plate must be oriented at $\theta = 45°$ relative to the input linear polarization. Input $\vec{E}_{\text{in}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$. For $\delta = \pi/4$, $B(d) = \begin{pmatrix} e^{-i\pi/4} & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$. The transformation matrix $M = R(-45°)B(d)R(45°)$ with $\theta = 45°$:

$$M = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} e^{-i\pi/4} & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$$

$$M = \frac{1}{2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} e^{-i\pi/4} & e^{-i\pi/4} \\ -e^{i\pi/4} & e^{i\pi/4} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} e^{-i\pi/4} + e^{i\pi/4} & e^{-i\pi/4} - e^{i\pi/4} \\ e^{-i\pi/4} - e^{i\pi/4} & e^{-i\pi/4} + e^{i\pi/4} \end{pmatrix}$$

Using Euler's formula ($e^{ix} = \cos x + i\sin x$): $e^{-i\pi/4} = \frac{1-i}{\sqrt{2}}$, $e^{i\pi/4} = \frac{1+i}{\sqrt{2}}$. $e^{-i\pi/4} + e^{i\pi/4} = 2\cos(\pi/4) = 2/\sqrt{2} = \sqrt{2}$. $e^{-i\pi/4} - e^{i\pi/4} = -2i\sin(\pi/4) = -2i/\sqrt{2} = -i\sqrt{2}$.

$$M = \frac{1}{2}\begin{pmatrix} \sqrt{2} & -i\sqrt{2} \\ -i\sqrt{2} & \sqrt{2} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix}$$

Applying to $\vec{E}_{\text{in}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$:

$$\vec{E}_{\text{out}} = M \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

This represents right-circularly polarized light (depending on convention). The thickness d required for $\delta = \pi/4$ is:

$$\frac{\pi}{4} = (n_s - n_f)\frac{\pi d}{\lambda} \implies d = \frac{\lambda}{4(n_s - n_f)}$$

For calcite and $\lambda = 600$ nm:

$$d = \frac{600 \times 10^{-9}\,\text{m}}{4 \times (0.172)} \approx 0.872 \times 10^{-6}\,\text{m} = 0.87\,\mu\text{m}$$

Answer: A quarter-wave plate ($\delta = \pi/4$) oriented at $\theta = 45°$ to the input polarization converts it to circular polarization. For calcite at $\lambda = 600$ nm, the required thickness is $d = \lambda/(4\Delta n) \approx 0.87\,\mu$m.

(c) Consider two linear polarizers oriented along the same direction, and a birefringent material placed between them. What is the transmitted intensity as a function of the orientation of the birefringent material relative to the axis of the polarizers?

Solution: Let the polarizers be oriented along the x-axis. The Jones matrix for such a polarizer is $L = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$[cite: 180]. Input light (unpolarized) passes through the first polarizer, resulting in $\vec{E}_1 = \begin{pmatrix} E_0 \\ 0 \end{pmatrix}$. This light then passes through the birefringent material oriented at angle θ. The Jones matrix for the material is $M(\theta, \delta) = R(-\theta)B(d)R(\theta)$. The light after the material is $\vec{E}_2 = M(\theta, \delta)\vec{E}_1$. Finally, the light passes through the second polarizer (also along x-axis): $\vec{E}_{\text{out}} = L\vec{E}_2 = LM(\theta, \delta)\vec{E}_1$. We need the (1,1) element of the matrix $M(\theta, \delta)$.

$$M(\theta, \delta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{-i\delta} & 0 \\ 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$M(\theta, \delta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} e^{-i\delta}\cos\theta & e^{-i\delta}\sin\theta \\ -e^{i\delta}\sin\theta & e^{i\delta}\cos\theta \end{pmatrix}$$

$$M_{11} = \cos\theta(e^{-i\delta}\cos\theta) - \sin\theta(-e^{i\delta}\sin\theta) = \cos^2\theta e^{-i\delta} + \sin^2\theta e^{i\delta}$$

$$M_{11} = \cos^2\theta(\cos\delta - i\sin\delta) + \sin^2\theta(\cos\delta + i\sin\delta)$$

$$M_{11} = (\cos^2\theta + \sin^2\theta)\cos\delta + i(-\cos^2\theta + \sin^2\theta)\sin\delta$$

$$M_{11} = \cos\delta - i\cos(2\theta)\sin\delta$$

The output electric field is:

$$\vec{E}_{\text{out}} = LM(\theta, \delta)\begin{pmatrix} E_0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix} \begin{pmatrix} E_0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} M_{11}E_0 \\ M_{21}E_0 \end{pmatrix} = \begin{pmatrix} M_{11}E_0 \\ 0 \end{pmatrix}$$

The transmitted intensity I_{out} is proportional to $|\vec{E}_{\text{out}}|^2 = |M_{11}E_0|^2 = |M_{11}|^2|E_0|^2$. Let the input intensity after the first polarizer be $I_0 \propto |E_0|^2$.

$$I_{\text{out}} = I_0|M_{11}|^2 = I_0|\cos\delta - i\cos(2\theta)\sin\delta|^2$$

$$I_{\text{out}} = I_0[(\cos\delta)^2 + (-\cos(2\theta)\sin\delta)^2]$$

$$I_{\text{out}} = I_0[\cos^2\delta + \cos^2(2\theta)\sin^2\delta]$$

Using $\sin^2\delta = 1 - \cos^2\delta$:

$$I_{\text{out}} = I_0[\cos^2\delta + \cos^2(2\theta)(1 - \cos^2\delta)]$$

$$I_{\text{out}} = I_0[\cos^2\delta + \cos^2(2\theta) - \cos^2(2\theta)\cos^2\delta]$$

$$I_{\text{out}} = I_0[\cos^2(2\theta) + \cos^2\delta(1 - \cos^2(2\theta))]$$

$$I_{\text{out}} = I_0[\cos^2(2\theta) + \cos^2\delta\sin^2(2\theta)]$$

Alternatively, using $\cos^2 x = (1 + \cos(2x))/2$:

$$I_{\text{out}} = I_0[\cos^2 \delta + \frac{1 + \cos(4\theta)}{2} \sin^2 \delta]$$

Let's re-derive using a simpler form: $M_{11} = \cos^2 \theta e^{-i\delta} + \sin^2 \theta e^{i\delta}$.

$$|M_{11}|^2 = (\cos^2 \theta e^{-i\delta} + \sin^2 \theta e^{i\delta})(\cos^2 \theta e^{i\delta} + \sin^2 \theta e^{-i\delta})$$

$$|M_{11}|^2 = \cos^4 \theta + \sin^4 \theta + \cos^2 \theta \sin^2 \theta(e^{-2i\delta} + e^{2i\delta})$$

$$|M_{11}|^2 = \cos^4 \theta + \sin^4 \theta + \cos^2 \theta \sin^2 \theta(2 \cos(2\delta))$$

$$|M_{11}|^2 = (\cos^2 \theta + \sin^2 \theta)^2 - 2 \cos^2 \theta \sin^2 \theta + 2 \cos^2 \theta \sin^2 \theta \cos(2\delta)$$

$$|M_{11}|^2 = 1 - 2 \cos^2 \theta \sin^2 \theta(1 - \cos(2\delta))$$

Using $2 \sin^2 \delta = 1 - \cos(2\delta)$ and $\sin(2\theta) = 2 \sin \theta \cos \theta$, so $2 \cos^2 \theta \sin^2 \theta = \frac{1}{2}(2 \sin \theta \cos \theta)^2 = \frac{1}{2} \sin^2(2\theta)$.

$$|M_{11}|^2 = 1 - \frac{1}{2} \sin^2(2\theta)(2 \sin^2 \delta) = 1 - \sin^2(2\theta) \sin^2 \delta$$

So,

$$I_{\text{out}} = I_0(1 - \sin^2(2\theta) \sin^2 \delta)$$

Checking consistency: $1 - \sin^2(2\theta) \sin^2 \delta = 1 - (1 - \cos^2(2\theta)) \sin^2 \delta = 1 - \sin^2 \delta + \cos^2(2\theta) \sin^2 \delta = \cos^2 \delta + \cos^2(2\theta) \sin^2 \delta$. This matches the previous form.

Answer: The transmitted intensity is $I_{\text{out}} = I_0(1 - \sin^2(2\theta) \sin^2 \delta)$, where I_0 is the intensity after the first polarizer, θ is the angle of the birefringent material's axes relative to the polarizers, and $\delta = (n_s - n_f)\pi d/\lambda$ is the phase retardation parameter.

Problem 14.4

What voltage must be applied across KDP to give a phase difference of π for 700 nm light?

Solution: The document provides the equation for the phase difference $\Delta\phi = \phi_x - \phi_y$ induced by an electric field E_z (voltage V across length l) in KDP (Eq. 13.30)[cite: 300]:

$$\Delta\phi = \frac{1}{c} \omega n_0^3 r_{63} E_z l = \frac{1}{c} \omega n_0^3 r_{63} V$$

where $\omega = 2\pi c/\lambda$ is the angular frequency of light.

$$\Delta\phi = \frac{1}{c} \left(\frac{2\pi c}{\lambda} \right) n_0^3 r_{63} V = \frac{2\pi}{\lambda} n_0^3 r_{63} V$$

We are given: $\Delta\phi = \pi$ $\lambda = 700\,\text{nm} = 700 \times 10^{-9}\,\text{m}$ [cite: 332] $n_0 = 1.51$ for KDP [cite: 298] $r_{63} = 10.6 \times 10^{-12}\,\text{m/V}$ for KDP [cite: 298]

We need to solve for the voltage V:

$$V = \frac{\Delta\phi\lambda}{2\pi n_0^3 r_{63}}$$

$$V = \frac{\pi \times (700 \times 10^{-9}\,\text{m})}{2\pi \times (1.51)^3 \times (10.6 \times 10^{-12}\,\text{m/V})}$$

$$V = \frac{700 \times 10^{-9}}{2 \times (3.443) \times (10.6 \times 10^{-12})}\,\text{V}$$

$$V = \frac{700 \times 10^{-9}}{73.0} \times 10^{12}\,\text{V} \approx 9.59 \times 10^3\,\text{V}$$

Answer: A voltage of approximately 9.6 kV must be applied across KDP.

Problem 14.5

This problem concerns an Acousto-Optic Modulator (AOM) using $LiNbO_3$. Given values: Material: $LiNbO_3$ [cite: 333] Light wavelength: $\lambda = 700\,\text{nm} = 7 \times 10^{-7}\,\text{m}$ [cite: 333] Light beam diameter: $D = 1\,\text{cm} = 0.01\,\text{m}$ [cite: 333] Acoustic power: $P_{\text{acoustic}} = 1\,\text{W}$ [cite: 333] Acoustic frequency: $\nu_s = 100\,\text{MHz} = 1 \times 10^8\,\text{Hz}$ [cite: 333] Acoustic beam dimensions: $1\,\text{mm} \times 1\,\text{mm}$[cite: 333]. Assume interaction length $l = 1\,\text{mm} = 0.001\,\text{m}$. Area $A_{\text{acoustic}} = (10^{-3}\,\text{m})^2 = 10^{-6}\,\text{m}^2$. $LiNbO_3$ properties[cite: 309]: Index of refraction: $n = 2.25$ Mass density: $\rho = 4700\,\text{kg/m}^3$ Sound velocity: $v_s = 7.40\,\text{km/s} = 7400\,\text{m/s}$ Photoelastic constant: $p = 0.15$

(a) What is the Bragg angle in the material?

Solution: The Bragg diffraction condition is given by Eq. (13.34)[cite: 305]:

$$2\lambda_{\text{phonon}} \sin\theta = \frac{\lambda_{\text{photon}}}{n}$$

where θ is the Bragg angle inside the material, $\lambda_{\text{phonon}} = v_s/\nu_s$ is the sound wavelength, and λ_{photon} is the light wavelength in vacuum.

$$\lambda_{\text{phonon}} = \frac{7400\,\text{m/s}}{1 \times 10^8\,\text{Hz}} = 7.4 \times 10^{-5}\,\text{m}$$

$$\sin\theta = \frac{\lambda_{\text{photon}}}{2n\lambda_{\text{phonon}}} = \frac{7 \times 10^{-7}\,\text{m}}{2 \times (2.25) \times (7.4 \times 10^{-5}\,\text{m})}$$

$$\sin\theta = \frac{7 \times 10^{-7}}{3.33 \times 10^{-4}} \approx 2.10 \times 10^{-3}$$

For small angles, $\sin\theta \approx \theta$ in radians.

$$\theta \approx 2.10 \times 10^{-3}\,\text{rad}$$

Converting to degrees: $\theta \approx (2.10 \times 10^{-3}) \times \frac{180}{\pi} \approx 0.12^\circ$

Answer: The Bragg angle inside the material is $\theta \approx 2.1 \times 10^{-3}$ radians or 0.12°.

(b) What is the ratio of diffracted to incoming intensity?

Solution: The ratio is given by Eq. (13.35)[cite: 307]:

$$\frac{I_{\text{diffracted}}}{I_{\text{incident}}} = \sin^2\left(\frac{\pi l}{\lambda_{\text{photon}}}\sqrt{\frac{MI_{\text{acoustic}}}{2}}\right)$$

First, calculate the acoustic intensity I_{acoustic}:

$$I_{\text{acoustic}} = \frac{P_{\text{acoustic}}}{A_{\text{acoustic}}} = \frac{1\,\text{W}}{10^{-6}\,\text{m}^2} = 1 \times 10^6\,\text{W/m}^2$$

Next, calculate the figure of merit M using Eq. (13.36)[cite: 308]:

$$M = \frac{n^6 p^2}{\rho v_s^3} = \frac{(2.25)^6(0.15)^2}{(4700\,\text{kg/m}^3)(7400\,\text{m/s})^3}$$

$$M = \frac{(113.9)(0.0225)}{(4700)(4.05 \times 10^{11})} = \frac{2.563}{1.90 \times 10^{15}} \approx 1.35 \times 10^{-15}\,\frac{\text{s}^3}{\text{kg}}$$

Now, calculate the argument of the $\sin^2$ function:

$$\text{Arg} = \frac{\pi l}{\lambda_{\text{photon}}}\sqrt{\frac{MI_{\text{acoustic}}}{2}}$$

$$\text{Arg} = \frac{\pi(0.001\,\text{m})}{7 \times 10^{-7}\,\text{m}}\sqrt{\frac{(1.35 \times 10^{-15})(1 \times 10^6)}{2}}$$

$$\text{Arg} = (4.49 \times 10^3)\sqrt{6.75 \times 10^{-10}} = (4.49 \times 10^3)(2.60 \times 10^{-5})$$

$$\text{Arg} \approx 0.117\,\text{rad}$$

Finally, the intensity ratio:

$$\frac{I_{\text{diffracted}}}{I_{\text{incident}}} = \sin^2(0.117) \approx (0.1167)^2 \approx 0.0136$$

Answer: The ratio of diffracted to incoming intensity is approximately 0.0136 or 1.36

(c) What sound frequency range is needed to resolve 1000 points?

Solution: The number of resolvable points (directions) N is given by Eq. (13.38)[cite: 314]:

$$N = \Delta\nu_s \tau$$

where $\Delta\nu_s$ is the sound frequency range (bandwidth) and τ is the time it takes sound to cross the light beam diameter D.

$$\tau = \frac{D}{v_s} = \frac{0.01\,\text{m}}{7400\,\text{m/s}} \approx 1.35 \times 10^{-6}\,\text{s} = 1.35\,\mu\text{s}$$

We need $N = 1000$.

$$\Delta\nu_s = \frac{N}{\tau} = \frac{1000}{1.35 \times 10^{-6}\,\text{s}} \approx 7.4 \times 10^8\,\text{Hz} = 740\,\text{MHz}$$

Answer: A sound frequency range (bandwidth) of approximately 740 MHz is needed to resolve 1000 points.